Exploration and Analysis of

Average Daily Apple Stock Price

Joseph Leong

Ryan Isherwood

Sam Whittaker

Statistics 457

November 2016

Performing analysis on time series is very important in our daily lives. It helps us make decisions regarding policy, lets us explore how our world works, and even helps with financial gain. The last option is of interest to us as we chose to analyze the mean price of Apple stock over the previous 10 year period. We wish to take this time series, render it stationary if it is not already, and fit an adequate ARIMA model to it. Once we believe we have found an adequate model for the data, we will use a few diagnostics to check and see if the model is indeed a proper fit. If we find that the model passes our tests, we will then perform a 10 day which we will then compare to the real values of the stock price to check if our model does a proper job of forecasting. In our conclusion we will discuss the benefits and drawbacks of using the ARIMA model along with other model options which may or may not do a better job of modelling the time series.

**The Data**:

Our data comes from the NASDAQ website and runs from November 9th 2006 to November 9th 2016. The observations were gathered on a daily basis, giving us a total of 2519 observations. With the data set containing so many observations it gives us the opportunity to observe not only the short run trends but all the long run trend which we will try to model using ARIMA. Figure 1 is a plot of the original untransformed data.



Figure 1: Plot of untransformed data

We can see that the plot exhibits a distinct upward trend which suggests that the data is not stationary. The plot also demonstrates non constant variance as there are times where the plot exhibits large volatility. One such point is between 2008 and 2009 as well as 2012 and 2013. This reinforces our assumption that the data is not stationary.



Figure 2: ACF and PACF of untransformed data

As we can see from figure 2, the autocorrelation function and partial autocorrelation function, hereafter referred to as the ACF and PACF respectively, again reinforce our assumptions that the data is not stationary and must be transformed to render it so. The ACF does not decrease to 0 at an exponential rate but rather at a linear one which means that we cannot model the data using ARIMA. We had some insight into using the launches of the Iphones as a possible seasonal component to the model as they all tend to be launched around the same time each year; however, we found that the effect they had was insignificant and thus we did not include it. Next we will go through the transformations performed on the data to make it stationary.

**Transformations**:

The first transformation we thought to try was a simple log transformation. We found that the plot shows an upward trend which once again suggested nonstationarity. This assumption is backed up by the ACF and PACF plots of the log transformed data. The ACF again shows that the correlation does not decrease to 0 at an exponential rate but again at a linear one so we had to try another transformation. The second transformation we tried was the log difference transformation which is often referred to as the growth rate. This is done by taking the first difference of the log transformed data.



Figure 3: Log difference transformed data

From figure 3 we can see a roughly stationary series that has a 0 mean and a constant variance with the exception of periods of volatility as outlined above. This makes sense as in theory the growth rate of financial assets is simply white noise with 0 mean and constant variance. Therefore, we conclude that the log difference transformation gives us a stationary series. We will next use the ACF and PACF of the transformed data, Figure 4, to get some insight as to which values of p and q we will use for our ARIMA(p,d,q) model. We already know that d will be 1 as we have taken the first difference of the data.



Figure 4: ACF and PACF of log difference transformed data

**Model Selection**:

For the model selection we first used the built in model selection tool in R which suggested we try a simple moving average model, MA(1). We tried this and found that the p-values from the Ljung-Box test for correlation are all within the rejection reason which lead us to believe there is still autocorrelation within the residuals. The residuals also did not seem to fit a normal distribution, thus we rejected this model. We used a loop to try all ARIMA(p,1,q) models with p=1,2,…,8 and q=1,2,…,8. We found three other models that seemed to be adequate fits based on their low AIC values and standard errors. The models suggested were the (2,1,3), (5,1,5), and (8,1,4) ARIMA models. The ARIMA(2,1,3) model had residuals that did not follow a normal distribution and were correlated according to the Ljung-Box test. The ARIMA(5,1,5) continued the pattern of non-normal residuals however for lower lag levels the residuals did seem to be uncorrelated. Next we tried the ARIMA(8,1,4) model and found it again had non-normal residuals and at higher lag levels they were correlated though at lags less than 10 we found that they were all uncorrelated according the Ljung-Box test. However, we noticed that the AR(7) and AR(8) coefficient values were not significant and we reduced the model to an ARIMA(6,1,4). After running the diagnostics again, we found that the residuals showed no correlation at lags less than 10. So we decided that this should be the model we will use for our time series. Unfortunately, we discovered that the function we were using to run the diagnostics for lower lag levels, the tsdiag() function, was flawed and we were not supposed to use it which let to the rejection of the ARIMA(6,1,4) model. So we decided to try and use the ACF and PACF of the log difference model to give us an idea of which values we should use for p and q as described in class and in the textbook. So after consulting Figure 4 we decided to try a (10,1,2) model and though the residuals did not follow an exact normal distribution which is seen in Figure 5 all lags aside from the first in the diagnostic had p-values well above the rejection region. This would suggest that the residuals are in fact uncorrelated which is contrary to our previous models. We then checked to see if the estimated coefficient values were significant by comparing them to twice their standard error. We discovered that even though a few of the auto regressive terms did not show significance, the highest ordered term did show significance and therefore we could not reject it.

|  |  |  |
| --- | --- | --- |
| ARIMA Model | AIC | Sigma^2 estimate |
| (0,1,1) | -133341.94 | 0.0002916 |
| (2,1,3) | -13347.93 | 0.0002899 |
| (5,1,5) | -13347.75 | 0.0002888 |
| (6,1,4) | -13349.54 | 0.0002885 |
| (10,1,2) | -13354.73 | 0.0002876 |

Table 1: AIC and sigma^2 estimates for all tested models

These facts combined with the lowest AIC value and smallest sigma^2 estimate of all models we tested meant that this would be the model we chose to perform our forecasting with.



Figure 5: Residual diagnostics for ARIMA(10,1,2)

**Our Final Model**:

(1+0.0904(0.0926)B+0.8194(0.0795)B2-0.1770(0.0312)B3+0.0128(0.0266)B4-0.0372(0.0261)B5-0.0024(0.0261)B6-0.0324(0.0261)B7+0.0617(0.0258)B8-0.0535(0.0215)B9+0.0666(0.0222)B10)(1-B)Xt=(1+0.3107(0.0914)B+0.8208(0.0694)B2)Wt

**Forecasting**:

We took the ARIMA(10,1,2) model, found above, and used it to perform a 10 day forecast. These values were then compared to the actual values, taken from the NASDAQ website, for those dates. We plotted the forecast along with the remaining data and found that the forecasted values were nearly invisible so we decided to include a subsection of the plot which includes only about 15 observations.



Figure 6: 10 day forecast using ARIMA(10,1,2)

As Figure 6 shows, the forecasted values demonstrate an upward trend which is to be expected since the data set as a whole shows a long run upward trend. The two lines on either side of the forecasted values are the 95% prediction intervals. Although they may seem constant, they do in fact grow larger as time passes. The reason them seem to be linear in nature is because the standard errors for our forecasted values are quite small and thus it would take many observations before we say any clear indication of non-linearity. We also wish to test how well our forecast does when compared to the actual price of the stock for our forecasted days. We can see in table 2 that our forecasted values are not all that far off from the actual values recorded on those days.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Date | Predicted value | Standard error | Actual value | % Error |
| 2016-11-11 | 110.3529 | 1.017 | 107.775 | 2.391 |
| 2016-11-14 | 110.4575 | 1.027 | 106.710 | 3.512 |
| 2016-11-15 | 110.6264 | 1.034 | 106.840 | 3.544 |
| 2016-11-16 | 110.7578 | 1.040 | 108.345 | 2.227 |
| 2016-11-17 | 110.9581 | 1.045 | 109.880 | 0.981 |
| 2016-11-18 | 110.9277 | 1.050 | 109.890 | 0.092 |
| 2016-11-21 | 111.1246 | 1.055 | 110.925 | 0.903 |
| 2016-11-22 | 111.2784 | 1.059 | 111.875 | 0.533 |
| 2016-11-23 | 111.2304 | 1.063 | 111.295 | 0.058 |
| 2016-11-24 | 111.3413 | 1.067 | 111.460 | 0.106 |

Table 2: Comparison of predicted value and actual values of next 10 days

Although the first 4 values do not fall within a 95% prediction interval, the remaining 6 fall well within the interval. We can also see that as time goes on, the amount of error in our prediction actually decreases which is counter intuitive. Normally we would see our errors get larger as time goes on since we do not know what will happen in the future and thus there is much more uncertainty in our prediction which would lead to larger errors. We believe that the reason for the decrease in error is due to fluctuations in the market following the US presidential election. If we look at the data prior to the 8th of November, we can see that the stock price is following an upward trend which is reflected in our predicted values. The price for the few days following the election decreases as the results of said election caused uncertainty in the market. Had the election not happened during our forecasting period, we believe that our prediction error would have been smaller for the first few observations and the actual observations would likely have fallen within our prediction interval.

**Conclusion**:

While our model has an adequate fit for our time series, it is not perfect. One of the reasons for this is likely due to the fact that the variance is not constant throughout the data which is not captured by an ARIMA model. To model this data better we may wish to try a GARCH model for modeling the volatility in our data set. We did try a GARCH model and though it did seem to do a better job of modelling the volatility and the data as a whole, we were unable to accurately use it for prediction. We may have chosen to use GARCH for our model if we had time during the semester to study it in detail; however, as we did not we have decided to omit it from the report. Our model may also be improved by looking at higher order ARIMA models. The problem with modelling with these higher order terms is that simply creating the model in R takes a very long time and may even crash the program itself. Given a sufficiently powerful processor we may be able to produce a higher order ARIMA model but we are limited by our personal computers and those provided to us. To conclude, yes our model is adequate but it can certainly be improved upon and should not be relied solely on to make any sort of financial decisions.

Citations:

NASDAQ. "Apple Inc. Common Stock (AAPL)." *NASDAQ.com*. NASDAQ, 01 Dec. 2016. Web. 02

Dec. 2016

Shumway, Robert H., and David S. Stoffer. *Time Series Analysis and Its Applications*. New York:

Springer, 2011. Print.